Exercise 3

I am only doing this one to train on the continuity equation!

a) The term $r(t)$ captures (similarly to an activity) the neurons that cross the threshold.

Those neurons spike and are reset, thus being re-introduced in a system that has cyclic boundary conditions.

b) x_0 is the neurons' reset potential! As neurons fire, their potential is reset to x_0.

c) The rate $r(t)$ measures how many neurons cross the threshold and fire. As such we have

$$r(t) = J(\theta, t)$$

d) The definition of $J(x, t)$ is FLAWED!

The independent variable x seems to be tied to t, as it only appears as $x(t)$.

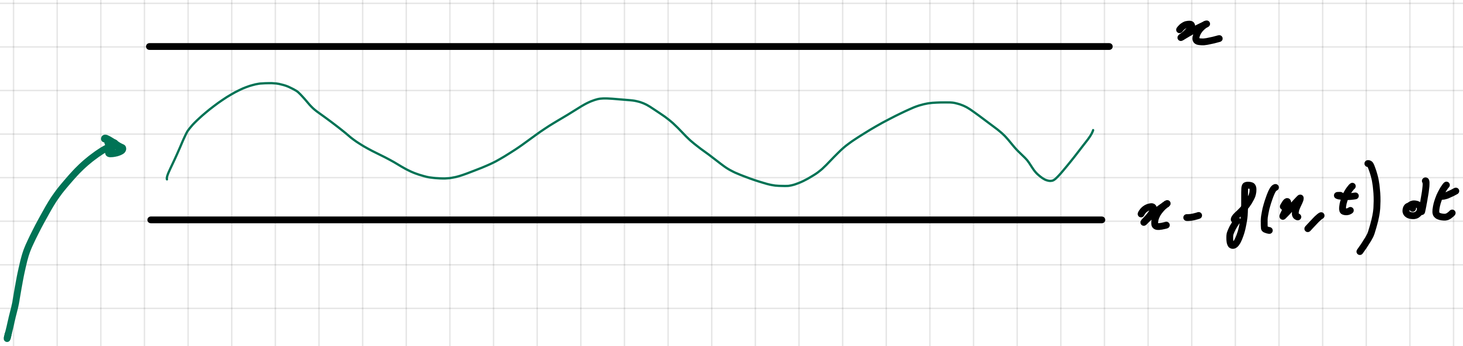
I will interpret "$x(t)$" as "x", hoping this is just a bad copy-paste.

There does not seem to be any noise here.
This looks like Leaky Integrate and Fire but let's figure this out properly.

Let us consider a neuron whose potential evolves as

$$\frac{dx}{dt} = f(x,t)$$

Now we want to compute the flow of time t through x.



All the neurons in here will cross the threshold within the next dt. Their number is

$$P(x,t)\, f(x,t)\, dt$$

In summary we have

$$J(x,t) = p(x,t)\, f(x,t)$$

which is satisfied by a neuron that behaves as

$$\frac{dx}{dt} = -x + x_1 + x_2\, e^{x/x_3} + x_4 \sin(\omega t)$$

So maybe this is... an exponential integrate-and-fire driven by a sinusoidal external current?

It seems so! From the slides of Week 1:

$$\tau \cdot \frac{d}{dt} u = F(u) + RI(t)$$

With

exponential I&F:

$$F(u) = -(u - u_{rest}) + c_0 \exp(u - \vartheta)$$

So in summary we have:

- Neuron model is an exponential integrate and fire. Resting potential is x_1.

- Noise is absent.

- the input is a sinusoidal drive

$$I_{ext}(t) = \frac{x_a}{R} \sin(\omega t)$$